HUDM4122
Probability and Statistical Inference

February 2, 2015

In the last class

- Covariance
- Correlation
- Scatterplots
- Simple linear regression

Questions? Comments?

Today

- Ch. 4.1-4.3 in Mendenhall, Beaver, & Beaver

Today

- Probability in Statistics
- Events and the Sample Space
- Calculating Probabilities Using Simple Events

I have a coin in my hand

- What is the probability that I will get heads?

I have a coin in my hand

- What is the probability that I will get heads?
  - 50%

I have a coin in my hand

- What is the probability that I will get tails?

I have a coin in my hand

- What is the probability that I will get tails?
  - 50%

Flipping the Coin

Now that I've flipped the coin

- What is the probability that I will get heads if I flip it again?

Flipping the coin 10 times

Flipping the coin 10 times

- What are the odds that I will get heads the next time?

Flipping the coin 10 times

- What are the odds that I will get heads the next time?

- It depends on whether it's a *fair coin*
  - E.g. a coin where the probability of heads is exactly 50%

But if you get heads 10 times in a row

- It’s probably not a *fair* coin

Probability of heads 1 time, if it’s a fair coin

- ½, or 50%, or 0.5

Probability of heads 2 times, if it’s a fair coin

Probability of heads 2 times, if it’s a fair coin

- 4 cases

- T, H
- H, T
- T, T
- H, H

Probability of heads 2 times, if it's a fair coin

- 4 cases

- T, H
- H, T
- T, T
- **H, H**

Probability of heads 2 times, if it's a fair coin

- 4 cases

- T, H
- H, T
- T, T
- **H, H**

- ¼ , or 25%, or 0.25

Probability of heads 3 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

Probability of heads 3 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

- 1/8 , or 12.5%, or 0.125

Probability of heads 4 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

Probability of heads 4 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

- 1/16 , or 0.0625

Probability of heads 5 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

Probability of heads 5 times, if it's a fair coin

- How many cases?
- How many of these cases are heads?

- 1/32, or 0.03125

Probability of heads 6 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

Probability of heads 6 times, if it's a fair coin

- How many cases?
- How many of these cases are heads?

- 1/64, or 0.015065

Probability of heads 7 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

Probability of heads 7 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

- 1/128, or 0.0075325

Probability of heads 8 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

Probability of heads 8 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

- 1/256, or about 0.0038

Probability of heads 9 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

Probability of heads 9 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

- 1/512, or about 0.002

Probability of heads 10 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

Probability of heads 10 times, if it’s a fair coin

- How many cases?
- How many of these cases are heads?

- 1/1024, or about 0.001

Who thinks this is a fair coin?

Who thinks this is a fair coin?

- It’s really improbable that it is
- About 1 in 1000

Who thinks this is a fair coin?

- It’s really improbable that it is
- About 1 in 1000
- Of course, weirder things have happened…
  - My daughter is 1 in 10,000,000,000, so…

Questions? Comments?

Quick non-on-the-test digression

We just did our first statistical test of the semester…

- The ***sign test***

We just did our first statistical test of the semester…

- The ***sign test***
  - A special case of it, anyways

The Sign Test

- Formulas and theory are out of this class's scope

The Sign Test

- Formulas and theory are out of this class's scope

- But the big idea is
- You have an event that you think happens 50% of the time
- But does it really occur 50% of the time?

End quick non-on-the-test digression

So, that coin

- Every time you flipped that coin it was a ***simple event***
  - An outcome that is received on a single repetition of an experiment

So, that coin

- Every time you flipped that coin it was a ***simple event***
  - An outcome that is received on a single repetition of an experiment

- So when I flipped the coin ten times, it represented 10 ***simple events***

So, that coin

- Every time you flipped that coin it was a ***simple event***
  - An outcome that is received on a single repetition of an experiment
- So when I flipped the coin ten times, it represented 10 ***simple events***
- An ***event*** is a set of connected ***simple events***

Mutual exclusivity

- When you and your significant other decide not to date anyone else?

Mutual exclusivity

- When you and your significant other decide not to date anyone else?

- No.

Mutual exclusivity

- When you and your significant other decide not to date anyone else?

- No. Not in stats class, anyways.

Mutual exclusivity

- ***Mutual Exclusivity*** is when two events cannot both occur together

- A coin cannot be both heads and tails at the same time

Mutual exclusivity

- ***Mutual Exclusivity*** is when two events cannot both occur together

- A coin cannot be both heads and tails at the same time

- Can you think of other examples of mutual exclusivity?

Questions? Comments?

Sample space

- The ***sample space*** is the set of all possible simple events

- So for a fair coin, it's {T, H}

What if you have a 6-sided die?

What if you have a 6-sided die?

- Sample space = {1, 2, 3, 4, 5, 6}

So if I have a six-sided die

- What is the probability that I roll a 6?

So if I have a six-sided die

- What is the probability that I roll a 6?
  - 1/6 or 0.17

So if I have a six-sided die

- What is the probability that I roll an odd number?

So if I have a six-sided die

- What is the probability that I roll an odd number?
  - 1, 3, 5
  - Out of
  - {1, 2, 3, 4, 5, 6}

So if I have a six-sided die

- What is the probability that I roll an odd number?
  - 1/2 or 0.5

So if I have a six-sided die

- What is the probability that if I roll the die twice, I get 1 both times?

So if I have a six-sided die

- What is the probability that if I roll the die twice, I get 1 both times?
  - 1/36

Questions? Comments?

Compound events

- Multiple simple events, one after another

What is the probability of getting 2 heads in a row

- T, T
- **H, H**
- T, H
- H, T

- 1 out of 4

More complex combinations possible too

Let’s look at another example

- Let’s say I flip a coin 4 times
- And I get 3 heads and 1 tail
  - We don’t care about the order

- What is the probability of that?

3 heads, 1 tail

- How many cases are there?

3 heads, 1 tail

- How many cases are there?

- 16
  - HHHH, HHHT, HHTH, HHTT,
  - HTHH, HTHT, HTTH, HTTT,
  - THHH, THHT, THTH, THTT,
  - TTHH, TTHT, TTTH, TTTT,

3 heads, 1 tail

- How many cases have 3 heads, 1 tail?

- 16
  - HHHH, HHHT, HHTH, HHTT,
  - HTHH, HTHT, HTTH, HTTT,
  - THHH, THHT, THTH, THTT,
  - TTHH, TTHT, TTTH, TTTT,

3 heads, 1 tail

- How many cases have 3 heads, 1 tail?

- 16
  - HHHH, **HHHT**, **HHTH**, HHTT,
  - **HTHH**, HTHT, HTTH, HTTT,
  - **THHH**, THHT, THTH, THTT,
  - TTHH, TTHT, TTTH, TTTT,

3 heads, 1 tail

- How many cases have 3 heads, 1 tail?

- 4/16 = 0.25
  - HHHH, **HHHT**, **HHTH**, HHTT,
  - **HTHH**, HTHT, HTTH, HTTT,
  - **THHH**, THHT, THTH, THTT,
  - TTHH, TTHT, TTTH, TTTT,

This is an illustration of probability

- To quote the book

- The ***probability of an event A*** is equal to the sum of the probabilities of the simple events contained in A

Example with 1 head out of 2 flips

- 4 cases
  - HH
  - ***HT*** (1/4)
  - ***TH*** (1/4)
  - TT

- ¼ + ¼ = ½

Example with 3 heads out of 4 flips

- 16 cases
  - HHHH, **HHHT**, **HHTH**, HHTT,
  - **HTHH**, HTHT, HTTH, HTTT,
  - **THHH**, THHT, THTH, THTT,
  - TTHH, TTHT, TTTH, TTTT,

- 1/16 + 1/16 + 1/16 + 1/16 = 4/16 = 1/4

Questions? Comments?

Probabilities don’t have to be equal for all examples in the sample space

Probabilities don’t have to be equal for all examples in the sample space

- For example, let’s say that you have made a new friend, randomly sampled from the population of New York City residents

Probabilities don’t have to be equal for all examples in the sample space

- For example, let’s say that you have made a new friend, randomly sampled from the population of New York City residents
  - That’s how I make friends

Probabilities don’t have to be equal for all examples in the sample space

- For example, let’s say that you have made a new friend, randomly sampled from the population of New York City residents
  - That’s how I make friends

- 56% of NYC residents take mass transit to work
- 28% of NYC residents drive to work

Probabilities don't have to be equal for all examples in the sample space

- For example, let's say that you have made a new friend, randomly sampled from the population of New York City residents
  - That's how I make friends

- 56% of NYC residents take mass transit to work
- 28% of NYC residents drive to work

- What is the probability that your friend either takes mass transit or drives to work?

Probabilities don't have to be equal for all examples in the sample space

- For example, let's say that you have made a new friend, randomly sampled from the population of New York City residents
  - That's how I make friends

- 56% of NYC residents take mass transit to work
- 28% of NYC residents drive to work

- What is the probability that your friend either takes mass transit or drives to work?
  - 84%

Questions? Comments?

If we have time

- Left-over from previous class

If we have time

- Demo finding A and B
  for $Y = A + Bx$

- In Excel

- Using Sum of Squared Residuals
  - Residual = Difference Between Predicted Y and Actual Y

Questions? Comments?

Upcoming Classes

- 2/9 No class
- 2/11 Permutations, Combinations, Unions, and Complements
  - Ch. 4.4
  - HW2 due
- 2/13 Independence and Conditional Probability
  - Ch. 4.5, 4.6

Homework 2

- Due in 7 days
- In the ASSISTments system

Questions? Comments?